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O‘ZARO DINAMIK BOG‘LANGAN G‘OVAK MUHITDA NEFT BOSIMINING G‘OVAKLIK KOEFFITSIENTIGA BOG‘LIQ O‘ZGARISHINI SONLI MODELLASHTIRISH

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Annotatsiya. Maqolada kam o‘tkazuvchan to‘siq orqali o‘zaro dinamik bog‘langan ikki qatlamli g‘ovak muhitda neftning nostatsionar filtratsiyasi uchun g‘ovaklik koeffitsientining bosimga bog‘liqligi m(p) va o‘tkazuvchanlikning Kozeni–Karman bog‘lanishini hisobga oluvchi matematik model taklif etilgan. Model oshkormas chekli ayirmali sxema va Pikar iteratsiyasi bilan yechildi. Hisoblash tajribalari boshlang‘ich g‘ovaklik 0,10 dan 0,30 gacha ortganda 720 sutkada quduqdagi bosim pasayishi 9,51 dan 6,15 MPa gacha (35 %) kamayishini, m(p) hisobga olinganda esa bosim pasayishi o‘zgarmas g‘ovaklik modeliga nisbatan 8–11 % ga katta bo‘lishini ko‘rsatdi.
Kalit so‘zlar: g‘ovak muhit, ko‘p qatlamli qatlam, qatlamlararo oqib o‘tish, g‘ovaklik koeffitsienti, deformatsiyalanuvchi kollektor, chekli ayirmalar usuli, sonli modellashtirish.

Аннотация. В статье предложена математическая модель нестационарной фильтрации нефти в двухслойной пористой среде, взаимно динамически связанной через слабопроницаемую перемычку, учитывающая зависимость пористости от давления m(p) и проницаемости от пористости по Козени–Кармана. Модель решена неявной конечно-разностной схемой с итерациями Пикара. Вычислительные эксперименты показали, что при росте начальной пористости от 0,10 до 0,30 депрессия на скважине за 720 сут уменьшается с 9,51 до 6,15 МПа (на 35 %), а учёт m(p) увеличивает депрессию на 8–11 % по сравнению с моделью постоянной пористости.
Ключевые слова: пористая среда, многослойный пласт, межпластовый переток, коэффициент пористости, деформируемый коллектор, метод конечных разностей, численное моделирование.

Abstract. The paper proposes a mathematical model of unsteady oil filtration in a two-layer porous medium mutually and dynamically coupled through a low-permeability barrier, accounting for pressure-dependent porosity m(p) and porosity-dependent permeability via the Kozeny–Carman relation. The model is solved by an implicit finite-difference scheme with Picard iterations. Computational experiments show that as the initial porosity increases from 0.10 to 0.30, the well drawdown after 720 days decreases from 9.51 to 6.15 MPa (by 35%), while accounting for m(p) increases the drawdown by 8–11% relative to the constant-porosity model.

Keywords: porous medium, multilayer reservoir, interlayer crossflow, porosity, deformable reservoir, finite difference method, numerical simulation.

KIRISH

Ko‘p qatlamli neft konlarida mahsuldor qatlamlar kam o‘tkazuvchan to‘siqlar bilan ajralgan bo‘lib, bosimlar farqi hisobiga ular orasida oqib o‘tish yuzaga keladi. Ikki muhit orasidagi almashinuv bosimlar farqiga proporsional degan g‘oya G.I. Barenblatt va hammualliflar tomonidan asoslangan [1] va ko‘p qatlamli neft tizimlariga tatbiq etilgan [4].

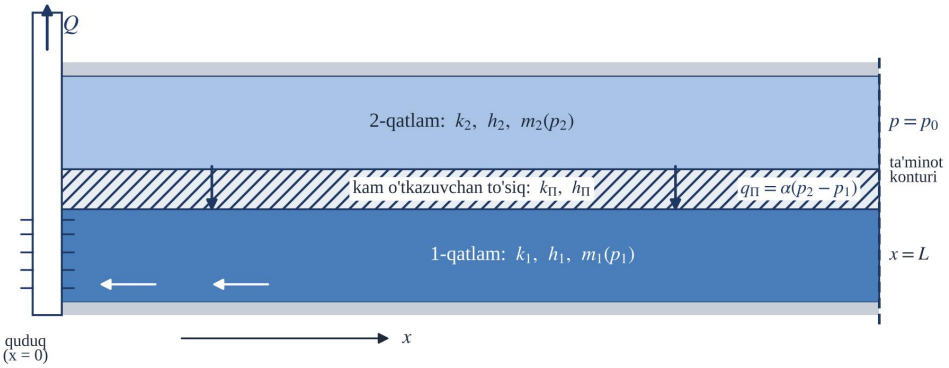
A. Nematov va hammualliflar uch qatlamli dinamik bog‘langan muhitda o‘tkazuvchanlik ortganda bosim tez tarqalib, quduqdagi pasayish sekinlashishini ko‘rsatgan [5]. I. Yuldoshev va hammualliflar Hantush modeli va moddiy balans modeli oralig‘idagi modelni hisoblash samaradorligi bo‘yicha optimal deb baholagan [6]. Sh. Kayumov va hammualliflar katta bosim gradienti zonasida oqib o‘tish keskin ortishini aniqlagan [7], K. Imanberdiyev va A. Karimov ikki qatlamni bitta quduq bilan ishlatish masalasini yechgan [8], B.Kh. Khuzhayorov va hammualliflar esa g‘ovak muhit xarakteristikalarining o‘zgarishini filtratsiya modeliga kiritgan [9].

Xalqaro ishlarda kollektor xossalarining bosimga bog‘liqligi asosan quduq sinovi uchun o‘rganiladi: J.-J. Guo va hammualliflar ko‘p qatlamli zich gaz qatlamlarida kuchlanishga sezgirlik asosan kechki davrda namoyon bo‘lishini [10], Q. Zhang va hammualliflar Pedrosa almashtirishi [2] asosida yarim analitik yechimni [11], Y. Zhang va hammualliflar qatlamlar farqi ortishi bilan bosim pasayishi kuchayishini [12] ko‘rsatgan.

Biroq bu ishlarda yo ko‘p qatlamli bog‘lanishda g‘ovaklik o‘zgarmas olingan [4–8], yo uning bosimga bog‘liqligi bitta qatlam uchun qaralgan [10, 11, 13]. Shu sababli ushbu ishning **maqsadi** — o‘zaro dinamik bog‘langan ikki qatlamli g‘ovak muhitda g‘ovaklik koeffitsientining bosimga bog‘liqligini hisobga olib, neft bosimining o‘zgarishini sonli modellashtirish va boshlang‘ich g‘ovaklik ta’sirini miqdoriy baholash.

TADQIQOT USULLARI

Fizik model. Uzunligi L va kengligi b bo‘lgan ikki mahsuldor qatlam qalinligi $h_П$ bo‘lgan kam o‘tkazuvchan to‘siq bilan ajratilgan (1-rasm). x = 0 da joylashgan quduq faqat 1-qatlamni ochgan va o‘zgarmas Q debit bilan ishlaydi, x = L da ta’minot konturi bosimi p_0 saqlanadi. Neft bir fazali va kuchsiz siqiluvchan, oqim izotermik va Darsi qonuniga bo‘ysunadi; qatlamlarda oqim gorizontal, to‘siqda vertikal deb olinadi [6, 7].



1-rasm. O‘zaro dinamik bog‘langan ikki qatlamli g‘ovak muhitning fizik sxemasi

Matematik model. G‘ovaklikning bosimga bog‘liqligi [13] dagi kabi eksponensial, o‘tkazuvchanlikning g‘ovaklikka bog‘liqligi esa Kozeni–Karman formulasi bilan beriladi:

$$m(p) = m_0 \exp[\beta_c (p - p_0)] \quad (1)$$

$$k(m) = k_0 \left(\frac{m}{m_0}\right)^3 \left(\frac{1-m_0}{1-m}\right)^2 \quad (2)$$

bu yerda m_0, k_0 — p_0 bosimdagi qiymatlar, β_c — g‘ovak hajmning siqiluvchanligi. Massaning saqlanish qonuni va Darsi qonunidan har bir qatlam uchun quyidagi tizim hosil bo‘ladi:

$$\beta^* m_1(p_1) \frac{\partial p_1}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_1(p_1)}{\mu}\frac{\partial p_1}{\partial x}\right] + \frac{\alpha}{h_1}(p_2 - p_1) \quad (3)$$

$$\beta^* m_2(p_2) \frac{\partial p_2}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_2(p_2)}{\mu}\frac{\partial p_2}{\partial x}\right] - \frac{\alpha}{h_2}(p_2 - p_1) \quad (4)$$

bu yerda $\beta^* = \beta_f + \beta_c$ — elastik sig‘im koeffitsienti, β_f — neftning siqiluvchanligi, μ — qovushoqlik, $\alpha = k_П/(\mu h_П)$ — qatlamlararo almashinuv koeffitsienti. Boshlang‘ich va chegaraviy shartlar:

$$p_i(x,0) = p_0, \quad \frac{k_1}{\mu}\frac{\partial p_1}{\partial x}\Big|_{x=0} = \frac{Q}{bh_1}, \quad \frac{\partial p_2}{\partial x}\Big|_{x=0} = 0, \quad p_i(L,t) = p_0 \quad (5)$$

(3)–(4) tizim m(p) va k(p) orqali bosimga nisbatan nochiziqli: g‘ovaklik pyezoo‘tkazuvchanlik $\chi = k/(\mu m \beta^*)$ ning ham surati (Kozeni–Karman orqali), ham maxrajiga kiradi.

Sonli usul. [0, L] oraliqda N = 201 tugunli tekis to‘r (h = 25 m) va τ = 1 sutka vaqt qadami olindi. Tenglamalar integro-interpolyatsion usul bilan oshkormas sxemaga keltirildi [14], tugunlararo o‘tkazuvchanlik garmonik o‘rtacha bilan hisoblandi. Nochiziqli koeffitsientlar Pikar iteratsiyasi bilan kvazichiziqlilashtirildi: (s+1)-iteratsiyada m va k qiymatlari $p^{(s)}$ bo‘yicha olinadi, ikkala qatlam tenglamalari almashinuv hadi bilan birga 2N noma’lumli bitta tizim sifatida yechiladi. Iteratsiya $\max|p^{(s+1)} - p^{(s)}|/p_0 < 10^{-9}$ bo‘lganda to‘xtatildi, har bir qadamda o‘rtacha 4 ta iteratsiya talab qilindi. O‘zgarmas g‘ovaklik holatida massa balansi xatosi 10^{-10} % dan kichik. Algoritm Python (NumPy, SciPy) tilida dasturlashtirildi, parametrlar 1-jadvalda keltirilgan.

1-jadval

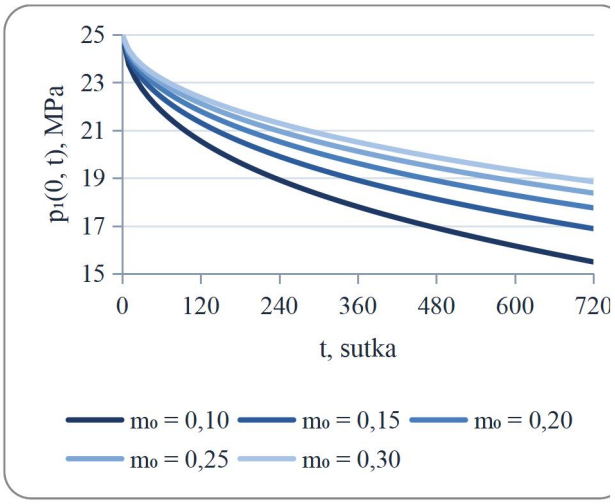
Hisoblash parametrlari

Parametr	Qiymati	Parametr	Qiymati
Qatlam uzunligi L	5000 m	Qovushoqlik μ	4 mPa·s
Qatlam kengligi b	1000 m	Neft siqiluvchanligi β_f	$1\cdot10^{-9}$ Pa^{-1}
Qalinliklar $h_1 = h_2$	10 m	G‘ovak hajm siqiluvchanligi β_c	$1\cdot10^{-8}$ Pa^{-1}
To‘siq qalinligi $h_П$	2 m	Boshlang‘ich bosim p_0	25 MPa
O‘tkazuvchanlik k_1 / k_2	5 / 3 mD	Quduq debiti Q	40 m^3/sut
To‘siq o‘tkazuvchanligi $k_П$	0,002 mD	Hisob muddati T	720 sut
Boshlang‘ich g‘ovaklik m_0	0,10–0,30	To‘r: N / τ	201 / 1 sut

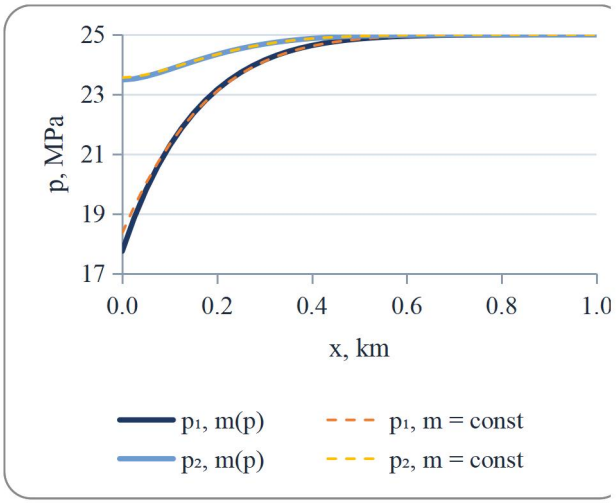
N = 51, 101, 201, 401 da $p_1(0, 720)$ mos ravishda 18,036; 17,810; 17,751; 17,736 MPa (m_0 = 0,20): yaqinlashish tartibi ≈ 1,9, N = 201 dagi xato 0,1 % dan kam; τ = 2 va 0,5 sutka natijani 0,002 MPa dan kam o‘zgartirdi.

NATIJALAR

2-rasmda m(p) hisobga olingan holda quduq tubidagi bosimning vaqt bo‘yicha o‘zgarishi berilgan. Boshlang‘ich g‘ovaklik qancha kichik bo‘lsa, bosim shuncha tez pasayadi: 180 sutkada $p_1(0, t)$ $m_0 = 0{,}10$ da 19,64 MPa, $m_0 = 0{,}30$ da 21,78 MPa; 720 sutkada esa mos ravishda 15,49 va 18,85 MPa. Sababi — kichik m_0 da qatlamning elastik zaxirasi kam, pyezoo‘tkazuvchanlik esa uch marta katta.



2-rasm. Quduq tubidagi bosim dinamikasi, m(p)



3-rasm. 720 sutkadagi bosim taqsimoti,$m_0 = 0{,}20$

3-rasmdan ko‘rinadiki, 2-qatlam quduqqa ochilmagan bo‘lsa ham, unda bosim x = 0 da 23,49 MPa gacha tushgan — bu oqib o‘tish orqali dinamik bog‘lanishning natijasi. m(p) hisobga olinganda 1-qatlam bosimi quduq atrofida pastroq yotadi: g‘ovaklik 0,20 dan 0,186 gacha kamayib, o‘tkazuvchanlik Kozeni–Karman bog‘lanishi bo‘yicha 22 % ga pasayadi. Barcha variantlar natijalari 2-jadvalda jamlangan.

2-jadval

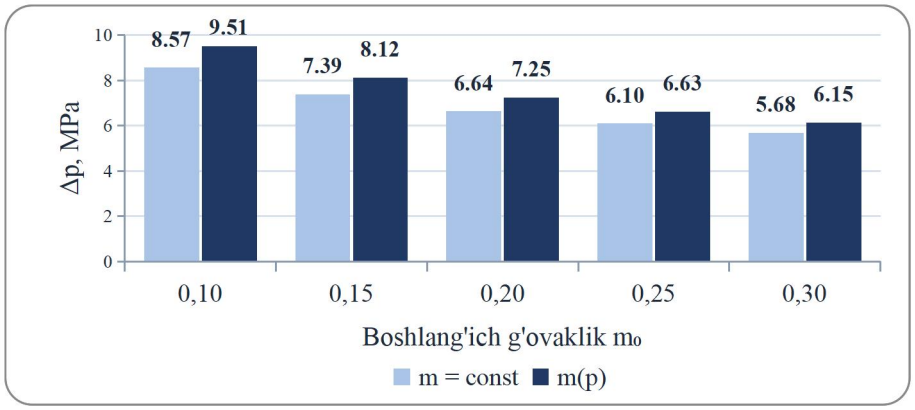
Boshlang‘ich g‘ovaklikning quduq ko‘rsatkichlariga ta’siri (x = 0)

m_0	p_1, MPa 180 sut	p_1, MPa 720 sut, m(p)	p_1, MPa 720 sut, m = const	Farq, MPa	Farq, %	m (720 sut)	k_1/k_0 (720 sut)	Oqib o‘tish ulushi, %
0,10	19,64	15,49	16,43	0,95	11,0	0,0909	0,737	47,5
0,15	20,53	16,88	17,61	0,72	9,8	0,1383	0,763	42,8
0,20	21,09	17,75	18,36	0,61	9,1	0,1860	0,777	38,1
0,25	21,49	18,37	18,90	0,53	8,7	0,2340	0,786	34,1
0,30	21,78	18,85	19,32	0,48	8,4	0,2821	0,790	30,7

Izoh: farq, % = $(p_{const} - p_{m(p)}) / (p_0 - p_{const}) \cdot 100$; oqib o‘tish ulushi — to‘siq orqali 1-qatlamga kelgan neftning Q ga nisbati, m(p) modeli.

2-jadval va 4-rasmga ko‘ra, m(p) hisobga olinganda bosim pasayishi barcha m_0 larda katta bo‘ladi: farq 0,48–0,95 MPa yoki o‘zgarmas g‘ovaklik modelidagi pasayishning 8,4–11,0 % ini tashkil etadi va m_0 kamayishi bilan ortadi. Bu ta’sir vaqt o‘tishi bilan kuchayadi: $m_0 = 0{,}20$ da farq 180 sutkada 0,18 MPa, 360 sutkada 0,34 MPa, 720 sutkada 0,61 MPa. G‘ovak hajm

siqiluvchanligi $\beta_c = 0{,}5\cdot10^{-8}$; $1\cdot10^{-8}$; $2\cdot10^{-8}$ Pa^{-1} bo‘lganda ($m_0 = 0{,}20$) farq mos ravishda 0,45; 0,61 va 0,79 MPa ni tashkil etdi.



4-rasm. 720 sutkadagi bosim pasayishi: o‘zgarmas g‘ovaklik va m(p) modellari

MUHOKAMA

Natijalar [5] dagi xulosani to‘ldiradi: o‘tkazuvchanlik ortishi quduqdagi bosim pasayishini sekinlashtirsa, bizning hisoblarda bosim pasaygan sari g‘ovaklik va u orqali o‘tkazuvchanlik kamayib, quduq atrofida o‘z-o‘zini kuchaytiruvchi bosim pasayishi yuzaga keladi. m(p) ta’sirining kechki davrda kuchayishi [10] dagi xulosaga, kichik g‘ovaklikda oqib o‘tish ulushining ortishi esa [12] natijasiga mos keladi.

Demak, past g‘ovakli deformatsiyalanuvchi kollektorlarda g‘ovaklikni o‘zgarmas deb olish quduq imkoniyatini oshirib baholaydi. Model bir o‘lchovli, bir fazali va izotermik; β_c kern tajribalaridan aniqlanishi, natijalar esa kon ma’lumotlari bilan tasdiqlanishi kerak.

XULOSA

1. Kam o‘tkazuvchan to‘siq orqali o‘zaro dinamik bog‘langan ikki qatlamli g‘ovak muhitda neft filtratsiyasining m(p) va k(m) bog‘lanishlarini hisobga oluvchi matematik modeli va oshkormas chekli ayirmali algoritmi ishlab chiqildi; algoritm ikkinchi tartibga yaqin yaqinlashishni ko‘rsatdi.

2. Boshlang‘ich g‘ovaklik 0,10 dan 0,30 gacha ortganda 720 sutkada quduqdagi bosim pasayishi 9,51 dan 6,15 MPa gacha (35 %) kamayadi, qatlamlararo oqib o‘tish ulushi esa 47,5 % dan 30,7 % gacha tushadi.

3. G‘ovaklikning bosimga bog‘liqligi bosim pasayishini 8,4–11,0 % ga oshiradi; bu ta’sir kichik g‘ovaklikda va ishlash muddati uzayishi bilan kuchayadi.

4. Model va dastur ko‘p qatlamli konlarda quduq rejimini tanlash hamda g‘ovaklikni o‘zgarmas deb olish xatosini baholash uchun qo‘llanilishi mumkin.

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