# BIR JINSLI BALKA TEBRANISH TENGLAMASI UCHUN ARALASH MASALA

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**Annotatsiya.** Ushbu ishda bir jinsli balka tebranish tenglamasi uchun aralash masala qo‘yilgan hamda uning yechimi mavjudligi va yagonaligi isbotlangan. Bu masalani yechishda o‘zgaruvchilarni ajratish ya’ni, Fure usulidan foydalanilgan. Topilgan yechimning va undan olingan kerakli hosilalarning yaqinlashuvchi ekanligi isbotlangan.
**Kalit so‘zlar:** balka tebranish tenglamasi, aralash masala Furye usuli, xos son, xos funksiya, Bessel tengsizligi, Furye koeffitsiyenti.

**Аннотация.** В данной работе поставлена смешанная задача для уравнения колебания однородной балки и докозаны существование и единственность её решения. При решении задачи применяется метод разделения переменных, то ест метод Фурье. Доказана сходимость найденного решения и необходимых производных, полученных из него.
**Ключевые слова:** уравнения колебания балки, смешанная задача, метод Фурье, собственное значение, неравенство Бесселя, коэффициент Фурье.

**Abstract.** In this paper, a mixed for the vibration equation of nonhomogeneous beam is formulated, and the existence and uniqueness of its solution are proved. The method of separation of variables, namely the Fourier method, is applied to solve the problem. The convergence of the obtained solution and its required derivatives is established.
**Keywords:** beam vibration equation , mixed problem, Fourier method, eigenvalue, eigenfunction, Bessel inequality, Fourier coefficient.

## Kirish

Matematik fizika tenglamalari orasida balka tebranish tenglamalari muhim o‘rin egallaydi va ular mexanika, qurilish hamda muhandislik sohalarida keng qo‘llaniladi. O‘zgaruvchan yoki murakkab tuzilishga ega bo‘lgan muhitlarda sodir bo‘ladigan mexanik va fizik jarayonlarni aniqlik bilan tasvirlashda to‘rtinchi tartibli operatorlar o‘ziga xos ahamiyat kasb etadi. Ushbu jarayonlarning fazo va vaqt davomidagi evolyutsiyasini o‘rganishda aralash masalalar – ya’ni fazoviy o‘zgaruvchilar bo‘yicha chegaraviy va vaqt bo‘yicha boshlang‘ich shartlar birgalikda beriladigan masalalar asosiy o‘rinni egallaydi. Ayniqsa, muhitning bir jinsli bo‘lmaganligi va fizik parametrlarning vaqt o‘zgaruvchanligi hisobga olinganda, mos matematik modellar murakkablashib boradi. Bunday hollarda tenglama koeffitsiyentlarining ayrim nuqtalarda nolga aylanishi yoki chegaraviy nuqtalarda buzilishi kuzatiladi, bu esa masalani tadqiq qilishni sezilarli darajada qiyinlashtiradi.

Bunday aralash masalalarni yechish va ularning xossalarini tahlil qilishning fundamental yondashuvlari XX asrning o‘rtalarida shakllanib, shundan buyon turli umumlashtirilgan holatlarda keng tadqiq qilinmoqda. Jumladan, V.S. Vladimirov [1], A.A. Samarskiy [2], S.L. Sobolev [3], V.A. Ilyin [4] va boshqa yetakchi olimlar klassik hamda noan’anaviy shartlar ostida xususiy hosilali differensial tenglamalar uchun aralash masalalarni chuqur o‘rganganlar.

**Asosiy qism**

Bir jinsli $p$ uzunlikka ega maxkamlangan balkani qaraymiz. Bu balkani majburiy ko‘ndalang tebranish tenglamasi, tashqi uzluksiz $G(x,t)$ kuch ta’sirida aylanma harakat bo‘lmagan holda quyidagicha yoziladi:

$$\rho S u_{tt} + EJ u_{xxxx} = G(x,t).$$

Bu yerda $\rho$ – balkani chiziqli zichligi, $S$ – ko‘ndalang kesim yuzasi, $E$ – materialning egiluvchanlik moduli, $J$ – o‘zini gorizontal o‘qi kesimida inertsiya momenti.

Bu tenglamani quyidagicha yozish mumkin:

$$u_{tt} + a^2 u_{xxxx} = f(x,t),$$

bu yerda

$$a^2 = EJ/(\rho S), \quad f(x,t) = G(x,t)/(\rho S).$$

Shuni ta’kidlash lozimki balkalar, sterjen va plastinlarni tebranish holatlari qurilish mexanikasida muhim rol o‘ynaydi. Biz xususiy holda $a = 1$, $f(x,t) = 0$ deb, olamiz. Keyinchalik $f(x,t)$ $0$ holni ham qaraymiz.

**Masalaning qo‘yilishi.**

$A$ **masala.** $D = \{(x,t): 0 < x < p, 0 < t < T\}$ sohada

$$u_{tt} + u_{xxxx} = 0 \qquad (1)$$

tenglamani qanoatlantiradigan hamda quyidagi

$$u(x,0) = \varphi_1(x) \qquad u_t(x,0) = \varphi_2(x) \qquad (2)$$

boshlang‘ich shartlarni va quyidagi

$$u(0,t) = 0 \qquad u(p,t) = 0, \quad u_{xx}(0,t) = 0 \quad u_{xx}(p,t) = 0 \qquad (3)$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ funksiya topilsin.

**Masala yechimining yagonaligi.**

**1-teorema.** Agar $A$ masalaning yechimi mavjud bo’lsa, u yagonadir.

**Isbot.** Teskarisidan faraz qilaylik. $A$ masalaning ikkita $u_1(x,t)$ va $u_2(x,t)$ yechimlarga ega bo’lsin. Bu yechimlarning ayirmasi bir jinsli (1) tenglamani va

(2)-(3) larga mos bir jinsli boshlang‘ich va chegaraviy shartlarni qanoatlantiradi. Bu ayirmani $u(x,t)$ bilan belgilaymiz, ya’ni

$$u(x,t) = u_1(x,t) - u_2(x,t) \qquad (4)$$

Ma’lumki,

$$X_n(x) = \sqrt{\frac{2}{p}} \sin \lambda_n x, \quad \lambda_n = \frac{\pi n}{p}, \quad n = 1,2,... \qquad (5)$$

funksiya $L_2(0,p)$ da to‘la ortonormallangan sistemani tashkil etadi.

Quyidagi

$$d_n(t) = \int_0^p u(x,t) X_n(x)\,dx \quad n = 1,2,... \qquad (6)$$

funksiyani qaraymiz. (6) ni quyidagi ko‘rinishda yozib olamiz:

$$d_{n\varepsilon}(t) = \int_{\varepsilon}^{p-\varepsilon} u(x,t) X_n(x)\,dx, 0 < \varepsilon < p, \qquad (7)$$

bu yerda $(\varepsilon, p-\varepsilon)$ .

(7) tenglikni t bo‘yicha ikki marta differensiallab,

$$d_{n\varepsilon}(t) = \int_{\varepsilon}^{p-\varepsilon} u_{tt}(x,t) X_n(x)\,dx$$

tenglikni hosil qilamiz. (1) tenglamani e’tiborga olib

$$d_{n\varepsilon}(t) = -\int_{\varepsilon}^{p-\varepsilon} u_{xxxx}(x,t) X_n(x)\,dx \qquad (8)$$

tenglikni hosil qilamiz. (8) tenglikni to‘rt marta bo’laklab integrallab, so‘ng $\varepsilon \to 0$ da limitga o‘tamiz:

$$\lim_{\varepsilon\to 0} d_{n\varepsilon}(t) = -\lim_{\varepsilon\to 0} X_n(p-\varepsilon)u_{xxx}(p-\varepsilon,t) - X_n(\varepsilon)u_{xxx}(\varepsilon,t) +$$
$$+\lim_{\varepsilon\to 0} X_n(p-\varepsilon)u_{xx}(p-\varepsilon,t) - X_n(\varepsilon)u_{xx}(\varepsilon,t) -$$
$$-\lim_{\varepsilon\to 0} X_n(p-\varepsilon)u_{x}(p-\varepsilon,t) - X_n(\varepsilon)u_{x}(\varepsilon,t) +$$
$$+\lim_{\varepsilon\to 0} X_n(p-\varepsilon)u(p-\varepsilon,t) - X_n(\varepsilon)u(\varepsilon,t) - \lim_{\varepsilon\to 0}\int_{\varepsilon}^{p-\varepsilon} X_n^{IV}(x)u(x,t)\,dx.$$

Oxirgi tenglikdan (3) ga mos bir jinsli shartlar hamda $X_n(x)$ funksiyaning xossalariga asosan quyidagi natija kelib chiqadi:

$$d_n(t) = -\lambda^4 \int_0^p u(x,t) X_n(n)dx \qquad n = 1,2,...$$

(4) ni e’tiborga olib quyidagi oddiy differensial tenglamaga kelamiz:

$$d_n(t) + \lambda^4 d_n(t) = 0, \qquad n = 1,2,... \qquad (9)$$

(2) shartga mos bir jinsli shartdan (6) ga asosan

$$d_n(0) = 0 \quad d_n(0) = 0 \quad n = 1,2,... \qquad (10)$$

boshlang‘ich shartlarni hosil qilamiz.

Ma’lumki, (9) oddiy differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda aniqlanadi:

$$d_n(t) = C_1 \cos\lambda^2 t + C_2 \sin\lambda^2 t$$

Bu yechimni (10) shartga bo‘ysundirib $C_1 = 0$ va $C_2 = 0$ ga ega bo‘lamiz, ya’ni barcha $\forall t$ $(0,t)$ uchun $d_n(t) = 0$. U holda (1.6) tenglik quyidagi ko‘rinishni oladi:

$$\int_0^p u(x,t) X_n(x)\,dx = 0, \; n = 1,2,... \qquad (11)$$

Ma’lumki, (5) to’la ortonormal sistemani tashkil etadi. (11) dan esa $u(x,t) = 0$ ekanligi kelib chiqadi. (4) ga asosan $u_1(x,t) = u_2(x,t)$ tenglik kelib chiqadi. Bundan ko‘rinadiki, agar $A$ masalaning yechimi mavjud bo‘lsa, u yagonadir. 1-teorema isbotlandi.

**Masala yechimining mavjudligi.**

**2-teorema.** Agar $\varphi(x)$ $C[0,1]$; $\varphi^{(k)}(x)$ $L(0,1)$ $\{k = 1,2,3,4,5\}$ bo‘lib

$$\varphi(0) = 0; \varphi(p) = 0; \quad \begin{matrix}\varphi_1(p) = \varphi_1(0) = 0 \\ \varphi_2(p) = \varphi_2(0) = 0\end{matrix}; \quad \begin{matrix}\varphi_1(p) = \varphi_1(0) = 0 \\ \varphi_2(p) = \varphi_2(0) = 0\end{matrix}; \quad \begin{matrix}\varphi_1^{IV}(p) = \varphi_1^{IV}(0) = 0 \\ \varphi_2^{IV}(p) = \varphi_2^{IV}(0) = 0\end{matrix}$$

tengliklar bajarilsa, u holda $A$ masalaning yechimi mavjud va u quyidagi ko‘rinishda aniqlanadi:

$$u(x,t)=\sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n}{p}\right)^2 t + b_n \sin\left(\frac{\pi n}{p}\right)^2 t \sqrt{\frac{2}{p}} \sin\frac{\pi n}{p}x, \qquad (12)$$

bu yerda

$$a_n=\sqrt{\frac{2}{p}}\int_0^p \varphi_1(x)\sin\frac{\pi n}{p}xdx,\ b_n=\sqrt{\frac{2}{p}}\frac{\lambda_n^2}{}\int_0^p \varphi_2(x)\sin\frac{\pi n}{p}xdx.$$

**Isbot:** $A$ masalaning yechimini mavjudligini isbotlash uchun o‘zgaruvchilarga ajratish (Furye) usulidan foydalanamiz. (1) tenglamaning yechimini quyidagi

$$u(x,t)=X(x)\ T(t) \qquad (13)$$

(13) ni (1) tenglamaga qo‘yib ushbu tenglikni hosil qilamiz:

$$\frac{X^{IV}(x)}{X(x)}=-\frac{T\ (t)}{T(t)}.$$

Bu tenglamaning chap tomoni faqat $x$ ga o‘ng tomoni esa faqat $t$ o’zgaruvchiga bog‘liqdir. Bundan kelib chiqadiki, tenglik o‘rinli bo‘lishi uchun ikki nisbat ham biror o‘zgarmas songa teng bo‘lishi kerak. Bu o‘zgarmasni $\lambda^4$ orqali belgilaymiz. Unda oxirgi tenglikdan, $X(x)$ va $T(t)$ noma’lum funksiyalarga nisbatan quyidagi oddiy differensial tenglamalar kelib chiqadi:

$$\begin{aligned} &X^{(IV)}(x)-\lambda^4 X(x)=0,\\ &T\ (t)+\lambda^4 T(t)=0. \end{aligned} \qquad (14)$$

Avvalo $X(x)$ ga nisbatan tenglamani yechib olaylik. Biz

$$X^{(IV)}(x)-\lambda^4 X(x)=0$$

tenglama va (3) chegaraviy shartlardan esa

$$X(0)=X(p)=X\ (0)=X\ (p)=0$$

chegaraviy shartlarga ega bo‘lamiz.

Bu tenglamaning umumiy yechimini topish uchun avvalo

$$X(x)=e^{kx}$$

deb belgilash kiritamiz bundan

$$k^4-\lambda^4=0$$

tenglik hosil bo‘ladi. Bunda $\lambda$ ning 3 xil holini ko‘rib chiqaylik:

1) $\lambda=0$ da $k^4=0$ bo’lib $k_{1,2,3,4}=0$ karrali yechimlarga ega bo‘lamiz. Bundan

$$X(x)=C_1+C_2x+C_3x^2+C_4x^3$$

yechim kelib chiqadi. Yechimni chegaraviy shartlarga bo‘ysundirsak

$$X(0)=0 \quad C_1=0, \qquad X\ (0)=0 \quad C_3=0,$$

$$X(p)=0 \quad C_2+C_3p+C_4p^2=0, \quad X\ (p)=0 \quad C_4=0 \quad C_2=0.$$

Demak,

$$X(x)=0.$$

Bu trivial yechim. Shuning uchun bu hol bizga kerak emas.

2) $\lambda>0$ da $k^4-\lambda^4=0$ bo’lib $k_{1,2}=\pm\lambda; k_{3,4}=\pm\lambda i$ yechimlar hosil bo‘ladi.

$X(x)$ funksiya esa

$X(x) = C_1 e^{\lambda x} + C_2 e^{-\lambda x} + C_3 \cos \lambda x + C_4 \sin \lambda x$

ga teng bo‘ladi. Yechimni chegaraviy shartlarga bo‘ysundirsak,

$X(0) = 0 \quad C_1 + C_2 + C_3 = 0,$

$X(p) = 0 \quad C_1 e^{\lambda p} + C_2 e^{-\lambda p} + C_3 \cos \lambda p + C_4 \sin \lambda p = 0,$

$X\ (0) = 0 \quad C_1 \lambda^2 + C_2 \lambda^2 - C_3 \lambda^2 = 0,$

$X\ (p) = 0 \quad C_1 \lambda^2 e^{\lambda p} + C_2 \lambda^2 e^{-\lambda p} - C_3 \lambda^2 \cos \lambda p - C_4 \lambda^2 \sin \lambda p = 0,$

hosil bo‘ladi. Sistemaga ko‘ra $C_1 = C_2 = C_3 = 0$ va $C_4 \sin \lambda p = 0$ lar kelib chiqadi. $C_4$ ni noldan farqli desak

$\sin \lambda p = 0$

bo‘ladi. Bundan

$\lambda p = \pi n, \quad \lambda = \frac{\pi n}{p}, n \quad N.$

$X\left(x\right)$ funksiya ortonarmalligidan esa $C_4 = \sqrt{\frac{2}{p}}$. Demak,

$X(x) = \sqrt{\frac{2}{p}} \sin \frac{\pi n}{p} x.$

3) $\lambda < 0$. Bunda $k^4 - \lambda^4 = 0$ bo’lib $k_{1,2} = \pm\lambda; k_{3,4} = \pm\lambda i$ lar 2-hol bilan ustma ust tushadi va $\lambda = \frac{\pi n}{p}$ xos son hosil bo‘ladi. $p$ musbat son bo‘lgani uchun $\lambda$ ham musbat bo‘lishi kerak bu esa $\lambda < 0$ ga zid. Shuning uchun bu holda yechim yo‘q.

Shunday ekan bizda faqatgina

$$X(x) = \sqrt{\frac{2}{p}} \sin \frac{\pi n}{p} x \qquad (15)$$

yechim mavjud.

Endi $T\left(t\right)$ ga bog‘liq differensial tenglamani qaraylik. Bunda ham huddi yuqoridagi ishlarni birin ketin bajarib chiqamiz va so‘ngida $T\left(t\right)$ ning umumiy yechimi quyidagicha chiqadi:

$T(t) = a_n \cos \lambda^2 t + b_n \sin \lambda^2 t$

$\lambda$ ni o‘rniga $\lambda = \frac{\pi n}{p}$ olib kelib qo‘ysak

$$T(t) = a_n \cos \left(\frac{\pi n}{p}\right)^2 t + b_n \sin \left(\frac{\pi n}{p}\right)^2 t, \qquad (16)$$

ga ega bo‘lamiz. $T(t)$ va $X(x)$ ning yechimlarini (13) ga olib kelib qo‘ysak biz qidirayotgan $A$ masalaning yechimi $u(x,t)$ hosil bo‘ladi va u quyidagicha ifodalanadi:

$u(x,t) = \sum_{n=1} a_n \cos \left(\frac{\pi n}{p}\right)^2 t + b_n \sin \left(\frac{\pi n}{p}\right)^2 t \sqrt{\frac{2}{p}} \sin \frac{\pi n}{p} x,$

bu yerda

$a_n = \sqrt{\frac{2}{p}} \int_0^p \varphi_1(x) \sin \frac{\pi n}{p} x dx \quad b_n = \sqrt{\frac{2}{p}} \lambda_n^2 \int_0^p \varphi_2(x) \sin \frac{\pi n}{p} x dx.$

**Lemma.** $\forall t \quad [0,T]$ va $n=1,2,...$ lar uchun quyidagi baholashlar o‘rinli:

$$|T_n(t)| \quad |a_n|+|b_n|, \quad |T_n(t)| \quad \lambda_n^2|a_n|+\lambda_n^2|b_n|, \quad |T_n(t)| \quad \lambda_n^4|a_n|+\lambda_n^4|b_n|. \qquad (17)$$

**Isbot.** (16) dan quyidagiga ega bo‘lamiz:

$$|T(t)|=\left|a_n \cos \frac{\pi n}{p}^2 t + b_n \sin \frac{\pi n}{p}^2 t\right| \quad |a_n+b_n| \quad |a_n|+|b_n|.$$

(16) dan quyidagi

$$T_n(t)= \frac{\pi n}{p}^2 a_n \cos \frac{\pi n}{p}^2 t + \frac{\pi n}{p}^2 b_n \sin \frac{\pi n}{p}^2 t$$

ifodani topib, so‘ngra uni baholaymiz:

$$|T_n(t)|=\left| \frac{\pi n}{p}^2 a_n \cos \frac{\pi n}{p}^2 t + \frac{\pi n}{p}^2 b_n \sin \frac{\pi n}{p}^2 t\right| \quad |a_n|\left|\cos \frac{\pi n}{p}^2 t\right| \frac{\pi n}{p}^2 +|b_n|\left|\sin \frac{\pi n}{p}^2 t\right| \frac{\pi n}{p}^2$$

$$\left| \frac{\pi n}{p}^2 \right||a_n|+\left| \frac{\pi n}{p}^2 \right||b_n|=\lambda_n^2|a_n|+\lambda_n^2|b_n|.$$

Endi (16) dan $T_n(t)$ ni quyidagi ko‘rinishda topamiz:

$$T_n(t)= \frac{\pi n}{p}^4 a_n \cos \frac{\pi n}{p}^2 t + \frac{\pi n}{p}^4 b_n \sin \frac{\pi n}{p}^2 t \ .$$

$T_n(t)$ funksiya uchun quyidagilar o‘rinli:

$$|T_n(t)|=\left| \frac{\pi n}{p}^4 a_n \cos \frac{\pi n}{p}^2 t + \frac{\pi n}{p}^4 b_n \sin \frac{\pi n}{p}^2 t\right| \quad |a_n|\left|\cos \frac{\pi n}{p}^2 t\right| \frac{\pi n}{p}^4 +|b_n|\left|\sin \frac{\pi n}{p}^2 t\right| \frac{\pi n}{p}^4$$

$$\left| \frac{\pi n}{p}^4 \right||a_n|+\left| \frac{\pi n}{p}^4 \right||b_n|=\lambda_n^4|a_n|+\lambda_n^4|b_n|,$$

Lemma isbotlandi.

Endi topilgan yechimni yaqinlashuvchi ekanligini va quyidagi

$$\frac{\partial^2 u}{\partial t^2}= \sum_{n=1} T_n(t) X_n(x), \qquad (18)$$

$$\frac{\partial^4 u}{\partial x^4}= \sum_{n=1} \lambda_n^4 T_n(t) X_n(x), \qquad (19)$$

qatorlarni yaqinlashuvchilikka tekshiramiz.

$A$ masalaning yechimini yaqinlashuvchilikka tekshirishdan boshlaylik.

(12) uchun

$$\sum_{n=1} |T_n(t)| \qquad (20)$$

qator mojarant qator bo‘ladi. lemmadagi (17) ni (20) ga qo‘yib yozsak,

$$\sum_{n=1} (|a_n|+|b_n|) \qquad (21)$$

hosil qilamiz. Endi (21) ga (13) tenglikni qo‘yib, quyidagi qatorni hosil qilamiz:

$$\sqrt{\frac{2}{p}}\sum_{n=1}^{}\int_0^p|\varphi_{1n}|+\sqrt{\frac{2}{p}}\left(\frac{p}{\pi n}\right)^2\sum_{n=1}^{}\int_0^p|\varphi_{2n}|, \qquad (22)$$

bu yerda

$$\varphi_{1n}=\int_0^p\varphi_1(x)\sin\frac{\pi n}{p}xdx,\quad \varphi_{2n}=\int_0^p\varphi_2(x)\sin\frac{\pi n}{p}xdx.$$

(22) qatorni yaqinlashuvchiligini ko‘rsatish maqsadida $\varphi_{1n}=\int_0^p\varphi_1(x)\sin\frac{\pi n}{p}xdx$ va $\varphi_{2n}=\int_0^p\varphi_2(x)\sin\frac{\pi n}{p}xdx$ ni bir marta bo‘laklab integrallaymiz:

$$\varphi_{1n}=\int_0^p\varphi_1(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}\left.\varphi_1(x)\cos\frac{\pi n}{p}x\right|_0^p-\int_0^p\varphi 1(x)\cos\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}\left(\varphi_1(p)\cos\pi n-\varphi_1(0)\cos 0\right)+\frac{p}{\pi n}\int_0^p\varphi_1(x)\cos\frac{\pi n}{l}xdx=$$

$$=\frac{p}{\pi n}\int_0^p\varphi_1(x)\cos\frac{\pi n}{p}xdx,$$

$$\varphi_{2n}=\int_0^p\varphi_2(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}\left.\varphi_2(x)\cos\frac{\pi n}{p}x\right|_0^p-\int_0^p\varphi_2(x)\cos\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}\left(\varphi_2(p)\cos\pi n-\varphi_2(0)\cos 0\right)+\frac{p}{\pi n}\int_0^p\varphi_2(x)\cos\frac{\pi n}{l}xdx=$$

$$=\frac{p}{\pi n}\int_0^p\varphi_2(x)\cos\frac{\pi n}{p}xdx.$$

2-teorema shartlarini e’tiborga olamiz va quyidagi tenglikni hosil qilamiz:

$$\varphi_{1n}=\frac{p}{\pi n}a_n,\ \varphi_{2n}=\frac{p}{\pi n}b_n, \qquad (23)$$

bu yerda

$$a_n=\int_0^p\varphi_1(x)\cos\frac{\pi n}{p}xdx,\quad b_n=\int_0^p\varphi_2(x)\cos\frac{\pi n}{p}xdx.$$

(23) ni (12)ga qo‘yib, o‘zgarmaslarni tashlab quyidagi natijaga kelamiz:

$$\sum_{n=1}\frac{|a_n|}{n}+\sum_{n=1}\frac{|b_n|}{n}. \qquad (24)$$

Bu qatorlarning yaqinlashuvchanligi elementar tengsizlikdan kelib chiqadi, ya’ni $\frac{|a_n|}{n}\ \frac{1}{2}\ a_n^2+\frac{1}{n^2}$, $\frac{|b_n|}{n}\ \frac{1}{2}\ b_n^2+\frac{1}{n^2}$,

va

$$\sum_{n=1} a_n^2, \quad \sum_{n=1} b_n^2, \quad \sum_{n=1} \frac{1}{n^2}.$$

Birinchi va ikkinchi qatorlarning yaqinlashuvchiligi bo‘lak-bo‘lak uzluksiz bo‘lgan $\varphi(x)$ funksiya uchun Bessel tengsizligidan kelib chiqadi. Ikkinchi qator umumlashgan garmonik $\sum_{n=1} \frac{1}{n^2}$ qator bo‘lib, u yaqinlashuvchidir. Demak, (24) qator tekis yaqinlashadi.

Shunday qilib biz (12) mojarant qatorning yaqinlashuvchiligini ko‘rsatdik. Demak, $D$ sohada (12) qator yani masalaning yechimi absolyut va tekis yaqinlashadi.

(18) uchun

$$\sum_{n=1} \left|T_n(t)\right| \qquad (25)$$

qator mojarant qator bo‘ladi. Lemmadagi (17) ni (25) ga qo‘yib yozsak,

$$\sum_{n=1} \lambda^4 |a_n| + \sum_{n=1} \lambda^4 |b_n| \qquad (26)$$

hosil qilamiz. Endi (18) ga (5) tenglikni qo‘yib quyidagi qatorni hosil qilamiz:

$$\left(\frac{\pi n}{p}\right)^4 \sqrt{\frac{2}{p}} \sum_{n=1} |\varphi_{1n}| + \left(\frac{\pi n}{p}\right)^4 \sqrt{\frac{2}{p}} \left(\frac{p}{\pi n}\right)^2 \sum_{n=1} |\varphi_{2n}|, \qquad (27)$$

bu yerda

$$\varphi_{1n} = \int_0^p \varphi_1(x) \sin\frac{\pi n}{p} x\,dx, \quad \varphi_{2n} = \int_0^p \varphi_2(x) \sin\frac{\pi n}{p} x\,dx.$$

(27) qatorni yaqinlashuvchiligini ko‘rsatish maqsadida $\varphi_{1n} = \int_0^p \varphi_1(x) \sin\frac{\pi n}{p} x\,dx$ va $\varphi_{2n} = \int_0^p \varphi_2(x) \sin\frac{\pi n}{p} x\,dx$ ni besh marta bo‘laklab integrallaymiz va 1-teoremadan foydalanib

$$\varphi_{1n} = \int_0^p \varphi_1(x) \sin\frac{\pi n}{p} x\,dx = -\frac{p}{\pi n} \varphi_1(x) \cos\frac{\pi n}{p} x \Big|_0^p - \int_0^p \varphi_1(x) \cos\frac{\pi n}{p} x\,dx =$$

$$= -\frac{p}{\pi n}\left(\varphi_1(p)\cos\pi n - \varphi_1(0)\cos 0\right) + \frac{p}{\pi n}\int_0^p \varphi_1(x)\cos\frac{\pi n}{p} x\,dx =$$

$$= \frac{p}{\pi n}\int_0^p \varphi_1(x)\cos\frac{\pi n}{p} x\,dx = \left(\frac{p}{\pi n}\right)^2 \varphi_1(x)\sin\frac{\pi n}{p} x \Big|_0^p - \int_0^p \varphi_1(x)\sin\frac{\pi n}{p} x\,dx =$$

$$= \left(\frac{p}{\pi n}\right)^2 \left(\varphi_1(p)\sin\pi n - \varphi_1(0)\sin 0\right) - \left(\frac{p}{\pi n}\right)^2 \int_0^p \varphi_1(x)\sin\frac{\pi n}{p} x\,dx =$$

$$= -\left(\frac{p}{\pi n}\right)^2 \int_0^p \varphi_1(x)\sin\frac{\pi n}{p} x\,dx = -\left(\frac{p}{\pi n}\right)^3 \varphi_1(x)\cos\frac{\pi n}{p} x \Big|_0^p - \int_0^p \varphi_1(x)\cos\frac{\pi n}{p} x\,dx =$$

$$= -\frac{p}{\pi n}^{3}\left(\varphi_1(p)\cos\pi n-\varphi_1(0)\cos 0\right)+\frac{p}{\pi n}^{3}\int_0^p \varphi_1(x)\cos\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{3}\int_0^p \varphi_1(x)\cos\frac{\pi n}{p}xdx=\frac{p}{\pi n}^{4}\varphi_1(x)\sin\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_1^{IV}(x)\sin\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{4}\left(\varphi_1(p)\sin\pi n-\varphi_1(0)\sin 0\right)-\frac{p}{\pi n}^{4}\int_0^p \varphi_1^{IV}(x)\sin\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}^{4}\int_0^p \varphi_1^{IV}(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}^{5}\varphi_1^{IV}(x)\cos\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_1^{V}(x)\cos\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{5}\int_0^p \varphi_1^{V}(x)\cos\frac{\pi n}{p}xdx,$$

$$\varphi_{2n}=\int_0^p \varphi_2(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}\varphi_2(x)\cos\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}\left(\varphi_2(p)\cos\pi n-\varphi_2(0)\cos 0\right)+\frac{p}{\pi n}\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=\frac{p}{\pi n}^{2}\varphi_2(x)\sin\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_2(x)\sin\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{2}\left(\varphi_2(p)\sin\pi n-\varphi_2(0)\sin 0\right)-\frac{p}{\pi n}^{2}\int_0^p \varphi_2(x)\sin\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}^{2}\int_0^p \varphi_2(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}^{3}\varphi_2(x)\cos\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}^{3}\left(\varphi_2(p)\cos\pi n-\varphi_2(0)\cos 0\right)+\frac{p}{\pi n}^{3}\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{3}\int_0^p \varphi_2(x)\cos\frac{\pi n}{p}xdx=\frac{p}{\pi n}^{4}\varphi_2(x)\sin\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_2^{IV}(x)\sin\frac{\pi n}{p}xdx=$$

$$=\frac{p}{\pi n}^{4}\left(\varphi_2(p)\sin\pi n-\varphi_2(0)\sin 0\right)-\frac{p}{\pi n}^{4}\int_0^p \varphi_2^{IV}(x)\sin\frac{\pi n}{p}xdx=$$

$$=-\frac{p}{\pi n}^{4}\int_0^p \varphi_2^{IV}(x)\sin\frac{\pi n}{p}xdx=-\frac{p}{\pi n}^{5}\varphi_2^{IV}(x)\cos\frac{\pi n}{p}x\Big|_0^p-\int_0^p \varphi_2^{V}(x)\cos\frac{\pi n}{p}xdx=$$

$$= \frac{p}{\pi n} \int_0^5 \varphi_2^V(x)\cos\frac{\pi n}{p}xdx.$$

Demak, $\varphi_{1n} = \frac{p}{\pi n}\int_0^5 \varphi_1^V(x)\cos\frac{\pi n}{p}xdx$ , $\varphi_{2n} = \frac{p}{\pi n}\int_0^5 \varphi_2^V(x)\cos\frac{\pi n}{p}xdx$ ekan. Bu tengliklardan quyidagi tengliklarni hosil qilamiz:

$$\varphi_{1n} = \frac{p}{\pi n}a_n,\ \varphi_{2n} = \frac{p}{\pi n}b_n, \qquad (28)$$

bu yerda

$$a_n = \int_0^p \varphi_1^V(x)\cos\frac{\pi n}{p}xdx,\ b_n = \int_0^p \varphi_2^V(x)\cos\frac{\pi n}{p}xdx.$$

(28) ni (27) ga qo‘yib, o‘zgarmaslarni tashlab quyidagi natijaga kelamiz:

$$\sum_{n=1}\frac{|a_n|}{n} + \sum_{n=1}\frac{|b_n|}{n}. \qquad (29)$$

Bu qatorlarning yaqinlashuvchanligi elementar tengsizlikdan kelib chiqadi, ya’ni $\frac{|a_n|}{n} \le \frac{1}{2}a_n^2 + \frac{1}{n^2}$ , $\frac{|b_n|}{n} \le \frac{1}{2}b_n^2 + \frac{1}{n^2}$ ,

va

$$\sum_{n=1} a_n^2,\ \sum_{n=1} b_n^2,\ \sum_{n=1}\frac{1}{n^2}.$$

Birinchi va ikkinchi qatorlarning yaqinlashuvchiligi bo‘lak-bo‘lak uzluksiz bo‘lgan $\varphi(x)$ funksiya uchun Bessel tengsizligidan kelib chiqadi. Ikkinchi qator umumlashgan garmonik $\sum_{n=1}\frac{1}{n^2}$ qator bo‘lib, u yaqinlashuvchidir. Demak, (29) qator tekis yaqinlashadi.

Shunday qilib biz (18) mojarant qatorining yaqinlashuvchiligini ko‘rsatdik. Demak, $D$ sohada (18) qator absolyut va tekis yaqinlashadi.

Endi (19) qatorning ham yaqinlashuvchiligini tekshiramiz. U uchun

$$\sum_{n=1}\lambda^4|T_n(t)| \qquad (30)$$

qator mojarant qator bo‘ladi. Lemmadagi (17) ni (30) ga qo‘yib yozsak,

$$\sum_{n=1}\lambda^4|a_n| + \sum_{n=1}\lambda^4|b_n|$$

yuqoridagi (26) tenglik hosil bo‘ladi. Demak, yuqoridagi ishlarni qayta takrorlasak (29) tenglikka kelamiz va o‘z navbztida bu qator yaqinlashuvchiligi bizga ma’lum. Bundan kelib chiqadiki (19) qator yaqinlashuvchi ekan. 2-teoremaning shartlari bajarilsa, unda $A$ masalaning $u(x,t)$ yechimini ikki marta $t$ bo‘yicha va to‘rt marta $x$ bo‘yicha hadlab differensiallanuvchi bo‘ladi, bundan kelib chiqadiki, topilgan qatorlar $D$ sohasida absolyut va tekis yaqinlashadi. 2-2-2-teorema isbotlandi.

**Xulosa**

Ushbu ishda bir jinsli balka tebranish tenglamasi uchun aralash masalaning yechimi mavjudligi va yagonaligi ko‘rib chiqilgan. Masala yechimi topilib undan turli baholar olingan va yaqinlashuvchilikka tekshirilgan. Masala yechimini topishda o‘zgaruvchilarga ajratish (Furye) usulidan foydalanilgan. Yaqinlashuvchiligini isbotlashda Bessel tengsizligi, umumlashgan garmonik qatorlardan foydalanilgan. Yechimning yagonaligi esa teskaridan faraz qilib topilgan.

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