# VAQTGA BOG'LIQ YUKLANISH OSTIDA QOVUSHOQELASTIK STERJENNING CHO'ZILISH JARAYONINI TADQIQ ETISH

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**Annotatsiya:** Mazkur maqolada vaqtga bog'liq yuklanish ostida qovushoqelastik sterjenning cho'zilish jarayoni nazariy jihatdan tadqiq qilinadi. Qovushoqelastik materiallarning mexanik xossalari elastik va qovushoqlik xususiyatlarining birgalikdagi ta'siri orqali ifodalanadi. Sterjenda hosil bo'ladigan deformatsiya va kuchlanishlarning vaqt bo'yicha o'zgarishi Kelvin–Voigt hamda Maxwell modellari yordamida tahlil qilinadi. Asosiy differensial tenglamalar chiqarilib, ularning fizik ma'nosi izohlanadi. Tadqiqot natijalari qovushoqelastik materiallarning vaqtga bog'liq deformatsion xususiyatlarini aniqlashda muhim ahamiyat kasb etadi.

**Kalit so'zlar:** qovushoqelastiklik, sterjen, cho'zilish, deformatsiya, kuchlanish, relaksatsiya, krip hodisasi, vaqtga bog'liq yuklanish.

**Kirish.**Hozirgi zamonaviy mashinasozlik, qurilish, aviatsiya va polimer texnologiyalarida qo'llanilayotgan ko'plab materiallar sof elastik emas, balki qovushoqelastik xossalarga ega hisoblanadi. Bunday materiallarda tashqi yuklanish ta'sirida deformatsiya faqat yuk miqdoriga emas, balki vaqt omiliga ham bog'liq bo'ladi. Shu sababli qovushoqelastik sterjenlarning cho'zilish jarayonini tadqiq qilish muhim ilmiy va amaliy masalalardan biri hisoblanadi. Qovushoqelastik materiallarda deformatsiya ikki asosiy mexanizm orqali yuz beradi:

1. Elastik deformatsiya;
2. Qovushoq oqim deformatsiyasi.

Elastik deformatsiya tashqi yuk olingandan keyin yo'qoladi, qovushoq deformatsiya esa vaqt davomida saqlanib qolishi mumkin.

Vaqtga bog'liq yuklanish ostida sterjenning mexanik holatini aniqlash uchun qovushoqelastiklik nazariyasining matematik modellaridan foydalaniladi.

Ko'ndalang kesimi doiraviy bo'lgan qovushoq-elastik sterjenning cho'zilish jarayoni uchun matematik modelini yaratamiz. Bunda, quyidagi asosiy fizik va matematik tushunchalar qo'llaniladi:

**Qovushoq-elastik material modeli:** Bu holatda Maxwell modeli yoki Kelvin–Voigt modeli ishlatiladi. Quyida Kelvin–Voigt modeli asosida cho'zilish jarayonining matematik modelini keltiramiz.

**Kelvin–Voigt modeli:** Bu model elastik va qovushoq elementlarni parallel ulangan holda tasvirlaydi.

$$\sigma(t) = E \cdot \varepsilon(t) + \eta \cdot \frac{d\varepsilon(t)}{dt} \qquad (1)$$

Bu yerda:

- $E$ - elastiklik moduli (Young moduli),
- $\eta$ - qovushoqlik koeffitsienti,
- $\sigma(t)$ - vaqtga bog'liq kuchlanish,
- $\varepsilon(t)$ - vaqtga bog'liq deformatsiya.

**Differensial tenglamasi.** Kuch ta'siri ostida quyidagicha differensial tenglama hosil bo'ladi:

$$\frac{F(t)}{A} = E \cdot \frac{u(t)}{l} + \frac{\eta}{l} \cdot \frac{du(t)}{dt} \qquad (2)$$

Bu tenglamani quyidagicha yozish mumkin:

$$F(t) = \frac{EA}{l} \cdot u(t) + \frac{\eta A}{l} \cdot \frac{du(t)}{dt} \quad (3)$$

**Yechish usuli.** Bu differensial tenglama berilgan F(t) bo‘yicha yechiladi. Masalan:

Agar $F(t)=F_0=const$, ya’ni doimiy kuch bo‘lsa:

$$\frac{\eta A}{l} \cdot \frac{du(t)}{dt} + \frac{EA}{l} \cdot u(t) = F_0 \quad (4)$$

Soddalashtirsak,

$$\frac{du(t)}{dt} + \frac{E}{\eta} \cdot u(t) = \frac{lF_0}{\eta A} \quad (5)$$

Bu oddiy birinchi tartibli differensial tenglama bo‘lib, umumiy yechimi:

$$u(t) = \frac{lF_0}{EA}\left(1 - e^{-\frac{E}{\eta}t}\right) \quad (6)$$

Agar cho‘zuvchi kuch $F(t)$ chiziqli o‘suvchi va so‘ng chiziqli kamayuvchi tarzda berilgan bo‘lsa, bu yuklama ikki bosqichli funksiyaga ega bo‘ladi. Quyida bu holat uchun **Kelvin–Voigt modeli** asosida cho‘zilish jarayonini matematik modellashtirish bosqichlarini amalga oshiramiz.

Faraz qilaylik, kuch quyidagi tarzda berilgan:

$$F(t) = \begin{cases} F_0 \frac{t}{t_1}, & 0 \le t \le t_1; \\ F_0\left(\frac{t_2}{t_2 - t_1} - \frac{t}{t_2 - t_1}\right), & t_1 < t \le t_2; \\ 0, & t > t_2 \end{cases} \quad (7)$$

Bu yerda:

- $F_0$ - maksimal kuch qiymati,
- $t_1$ - kuch o‘sishi tugaydigan vaqt,
- $t_2$ - kuch butunlay nolga tushadigan vaqt.

Oldingi javobda keltirilganidek, differensial tenglamani quyidagicha yozamiz:

$$\frac{du(t)}{dt} + \frac{E}{\eta} \cdot u(t) = \frac{l}{\eta A} F(t) \quad (8)$$

Har bir vaqt oralig‘i uchun differensial tenglama alohida yechiladi.

1. $0 \le t \le t_1$ uchun tenglamaning ko‘rinishi:

$$\frac{du(t)}{dt} + \frac{E}{\eta} \cdot u(t) = \frac{lF_0}{\eta A} \frac{t}{t_1} \quad (9)$$

Bu tenglamaning $u(0) = 0$ bo‘lgandagi yechimi quyidagicha:

$$u(t) = \frac{F_0 l}{EA}\left[t - \frac{\eta}{E}\left(1 - e^{-\frac{E}{\eta} \cdot t}\right)\right] \quad (10)$$

2. $t_1 < t \le t_2$ uchun tenglamaning standart ko‘rinishi:

3. $$\frac{du(t)}{dt} + \frac{E}{\eta} \cdot u(t) = \frac{lF_0}{\eta A}\left(\frac{t_2 - t}{t_2 - t_1}\right) \quad (11)$$

Tenglamaning umumiy yechimi quyidagicha

$$u(t) = \frac{F_0 l}{EA}\left(\frac{1}{t_2 - t_1}\right)\left[(t_2 - t) + \frac{\eta}{E}\right] + C \cdot e^{-\frac{E}{\eta} \cdot t} \quad (12)$$

Bunda $C$ ni

$$u(t_1) = \frac{F_0 l}{EA}\left[t_1 - \frac{\eta}{E}\left(1 - e^{-\frac{E}{\eta} \cdot t_1}\right)\right] \quad (13)$$

bo‘lgan holdan aniqlaymiz:

$$C = \frac{\frac{F_0 l}{EA}(t_1 - 1) - \frac{\eta}{E}\frac{F_0 l}{EA}\left[\frac{t_2 - t_1 - 1}{t_2 - t_1} e^{-\frac{E}{\eta} \cdot t_1}\right]}{e^{-\frac{E}{\eta} \cdot t_1}} \quad (14)$$

Yakuniy yechim

$$u(t) = \frac{F_0 l}{EA}\left(\frac{1}{t_2 - t_1}\right)\left[(t_2 - t) + \frac{\eta}{E}\right] + \left[\frac{F_0 l}{EA}(t_1 - 1) - \frac{\eta}{E} \cdot \frac{F_0 l}{EA}\left(\frac{t_2 - t_1 - 1}{t_2 - t_1} - e^{-\frac{E}{\eta} \cdot t_1}\right)\right] \cdot e^{-\frac{E}{\eta}(t - t_1)}. \quad (15)$$

4. $t>t_2$ uchun yuklama nolga tushganda: $F(t)=0$ tenglamaning ko‘rinishi:

$$\frac{du(t)}{dt}+\frac{E}{\eta}\cdot u(t)=0 \qquad (16)$$

Tenglamaning yechimi

$$u(t)=u(t_2)\cdot e^{-\frac{E}{\eta}(t-t_2)} \qquad (17)$$

Bu bosqichda deformatsiya asta-sekin nolga yaqinlashadi, lekin to‘liq nol bo‘lmaydi. Bu esa qovushoq-elastiklikning asosiy xususiyatidandir.

**Masala:** Sterjen materiali poliuretin $= 2\cdot 10^9\ Pa$ ; $\eta = 10^6\ Pa\cdot s$.Sterjen o‘lchamlari: $l = 20\ cm$; $r = 2\ cm$.

Sterjenga qo‘yilgan kuchni vaqtdan bog‘liq o‘zgarishi:

$F_0 = 60\ kN$; $t_1 = 5\ sek$; $t_2 = 10\ sek$;

$$\sigma=\frac{F}{A}$$

Maksimal kuchlanish:

$$\sigma_0=\frac{60000}{0.001256}$$

$$\sigma_0\approx 4.78\cdot 10^7\ Pa$$

Asosiy tenglama:

$$\sigma=E\varepsilon+\eta\frac{d\varepsilon}{dt}$$

$$\frac{F(t)}{A}=E\varepsilon+\eta\frac{d\varepsilon}{dt}$$

*yoki*

$$\frac{d\varepsilon}{dt}+\frac{E}{\eta}\varepsilon=\frac{F(t)}{A\eta}$$

$$\frac{d\varepsilon}{dt}+\frac{2\cdot 10^9}{10^6}\varepsilon=\frac{F(t)}{0.001256\cdot 10^6}$$

$$\frac{d\varepsilon}{dt}+2000\varepsilon=\frac{F(t)}{1256}$$

Yuk ortuvchi qism. Kuch funksiyasi:

$$F(t)=\frac{F_0}{t_1}t$$

$$F(t)=\frac{60000}{5}t$$

$$F(t)=12000t$$

$$\frac{d\varepsilon}{dt}+2000\varepsilon=\frac{12000t}{1256}$$

$$\frac{d\varepsilon}{dt}+2000\varepsilon=9.554t$$

$$\mu(t)=e^{2000t}$$

$$\varepsilon_1(t)=0.004777t-2.388\cdot 10^{-6}+Ce^{-2000t}$$

$$\varepsilon(0)=0$$

$$C=2.388\cdot 10^{-6}$$

$$\varepsilon_1(t)=0.004777t-2.388\cdot 10^{-6}+2.388\cdot 10^{-6}\cdot e^{-2000t}$$

t=5 sekunddagi deformatsiya

$$\varepsilon_1(5)=0.004777(5)-2.388\cdot 10^{-6}$$

$$e^{-10000}\approx 0$$

$$\varepsilon_1(5)\approx 0.02388$$

Yuk kamayuvchi qism

$$F(t)=60000\cdot\left(1-\frac{t-5}{5}\right)$$

$$F(t)=120000-12000t$$

$$\frac{d\varepsilon}{dt}+2000\varepsilon=95.54-9.554t$$

$$\varepsilon_2(t)=0.05255-0.004777t+C\cdot e^{-2000t}$$

$$\varepsilon_2(5)=0.02388$$

$$0.02388=0.05255-0.004777t+C\cdot e^{-10000}$$

$$0.02388\approx 0.02867+0$$

Eksponensial had juda kichik bo‘lgani uchun:

$$C\approx 0$$

Yuk nol bo‘lgandan keyin

$$F(t)=0$$

$$\frac{d\varepsilon}{dt}+2000\varepsilon=0$$

$$\varepsilon_3(t)=Ce^{-2000t}$$

$$\varepsilon_2(10)=0.05255-0.004777\cdot 10$$

$$\varepsilon_2(10)=0.00478$$

$$\varepsilon_3(t)=0.00478e^{-2000(t-10)}$$

$$\varepsilon(t)=\begin{cases}0.004777t-2.388\cdot 10^{-6}+2.388\cdot 10^{-6}\cdot e^{-2000t}, & 0\le t\le 5\\ 0.05255-0.004777t\ , & 5\le t\le 10\\ 0.00478e^{-2000(t-10)}, & t>10\end{cases}$$

Cho‘zilish formulasi:

$$\Delta L=\varepsilon L$$

$$\varepsilon_{max}\approx 0.02388$$

$$L=0.2\ m$$

$$\Delta L_{max}=0.02388\cdot 0.2$$

$$\Delta L_{max}\approx 4.78\ mm$$

**Xulosa** . Mazkur tadqiqotda vaqtga bog‘liq yuklanish ostida qovushoqelastik sterjenning cho‘zilish jarayoni Kelvin–Voigt modeli asosida nazariy va amaliy jihatdan tahlil qilindi. Tadqiqot davomida qovushoqelastik materiallarning elastik va qovushoq xususiyatlari birgalikda hisobga olinib, sterjenda hosil bo‘ladigan deformatsiyaning vaqt bo‘yicha o‘zgarish qonuniyatlari aniqlandi. Masalani yechish jarayonida sterjen materiali sifatida poliuretin tanlandi hamda uning mexanik parametrlari asosida differensial tenglamalar tuzildi. Yuklamaning vaqt bo‘yicha chiziqli ortishi va kamayishi holatlari uchun alohida analitik yechimlar olindi. Hisoblash natijalari shuni ko‘rsatdiki, yuk ortishi davomida sterjendagi deformatsiya ham ortib boradi, maksimal deformatsiya esa maksimal yuklanish qiymati yaqinida yuzaga keladi.

Tahlillar natijasida sterjenning maksimal nisbiy deformatsiyasi:

$$\varepsilon_{max}\approx 0.02388$$

ekanligi aniqlandi. Sterjenning maksimal uzayishi esa:

$$\Delta L_{max}\approx 4.78\ mm$$

ga teng bo‘ldi.

Yuk olib tashlangandan keyin deformatsiyaning birdan yo‘qolmasligi, balki eksponensial qonun asosida asta-sekin kamayishi qovushoqelastik materiallarning asosiy fizik xususiyatini tasdiqladi. Tadqiqot natijalari Kelvin–Voigt modeli qovushoqelastik sterjenlarning vaqtga bog‘liq deformatsiyasini tavsiflashda samarali model ekanligini ko‘rsatdi.

Olingan nazariy va amaliy natijalar polimer materiallar, amortizatsion elementlar, kompozit konstruksiyalar hamda zamonaviy muhandislik tizimlarini loyihalash va mustahkamlikka hisoblash masalalarida muhim ahamiyatga ega.

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